SPECIAL QUESTION PAPER 2016

Class: XIITHSTD Subject: **MATHEMATICS** Marks: 200 Time: 3 : 00 hrs.

SECTION - A (40 x 1 = 40)

(i) Answer all the questions. (ii) Choose and write the correct answer.

1. The rank of the matrix is $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$ is
   (a) 2 (b) 1 (c) 3 (d) 4
2. If I is the unit matrix of order n, where k ≠ 0 is a constant, then adj(kI)
   (a) k^n(adj$I$) (b) k(adj$I$) (c)k^2(adj$I$) (d)k^{n-1} (adj$I$)
3. If A and B are matrices conformable to multiplication then $(AB)^T$ is
   (a) $A^T B^T$ (b) $B^T A^T$ (c) AB (d) BA
4. p (A) ≠ p [A, B] then the system is
   (a) consistent and has infinitely many solution (b) consistent and has unique solution (c) consistent (d) inconsistent
5. The area of the parallelogram having a diagonal $3\vec{i} + \vec{j} - \vec{k}$ and a side $\vec{i} - 3\vec{j} + 4\vec{k}$ is
   (a) $10\sqrt{3}$ (b) $6\sqrt{30}$ (c) $\frac{3}{2}\sqrt{30}$ (d) $3\sqrt{30}$
6. The following two lines are $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z-1}{2}$
   (a) parallel (b) intersecting (c) skew (d) perpendicular
7. The angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$ is
   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$
8. The non-parametric vector equation of a plane passing through a point whose P.V is $\vec{a}$ and parallel to $\vec{u}$ and $\vec{v}$ is
   (a) $[\vec{r} - \vec{a}, \vec{u}, \vec{v}] = 0$ (b) $[\vec{r}, \vec{u}, \vec{v}] = 0$ (c) $[\vec{r}\vec{a}\vec{u} \times \vec{v}] = 0$ (d) $[\vec{a}\vec{u}\vec{v}] = 0$
9. If $m\vec{i} + 2\vec{j} + \vec{k}$ and $4\vec{i} - 9\vec{j} + 2\vec{k}$ are perpendicular then m is
   (a) - 4 (b) 8 (c) 4 (d) 12
10. If $\vec{a}, \vec{b}, \vec{c}$ are a right handed triad of mutually perpendicular vectors of magnitude a, b, c then the value of $[\vec{a}\vec{b}\vec{c}]$ is
    (a) $a^2 b^2 c^2$ (b) 0 (c) $\frac{1}{2}abc$ (d) abc
11. If $z_n = \cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}$ then $z_1 z_2 \ldots z_6$ is
    (a) 1 (b) - 1 (c) i (d) - i
12. If the amplitude of a complex number is $\pi/2$ then the number is
    (a) Purely imaginary (b) purely real (c) 0 (d) neither real nor imaginary
13. The number of values of $(\cos\theta + \sin\theta)^{p/q}$ where p and q are non-zero integers prime to each other is
    (a) p (b) q (c) p + q (d) p - q
14. If ω is a cube roots of unity then
    (a) $\omega^2 = 1$ (b) $1 + \omega = 0$ (c) $1 + \omega + \omega^2 = 0$ (d) $1 + \omega - \omega^2 = 0$
15. The vertex of the parabola $x^2 = 8y - 1$ is

(a) $\left(-\frac{1}{8},0\right)$ (b) $\left(\frac{1}{8},0\right)$ (c) $\left(0,\ \frac{1}{8}\right)$ (d) $\left(0,\ \frac{1}{8}\right)$

16. The asymptotes of the hyperbola $36y^2 - 25x^2 + 900 = 0$ are

(a) $y = \pm\frac{6}{5}x$ (b) $y = \pm\frac{5}{6}x$ (c) $y = \pm\frac{36}{25}x$ (d) $y = \pm\frac{25}{36}x$

17. The equations of the major and minor axes of $4x^2 + 3y^2 = 12$ are

(a) $x = \sqrt{3}$, $y = 0$ (b) $x = 0$, $y = 0$ (c) $x = -\sqrt{3}$, $y = 0$ (d) $x = 0$, $y = 0$

18. The equation of chord of contact of tangents from the (2, 4) to the ellipse $2x^2 + 5y^2 = 20$is

(a) $x - 5y + 5 = 0$ (b) $5x - y + 5 = 0$ (c) $x + 5y - 5 = 0$ (d) $5x - y - 5 = 0$

19. For the curve $x = e^t\cos t$; $y = e^t \sin t$ the tangent line is parallel to the x- axis when t is equal to

(a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{\pi}{2}$

20. $\lim_{x\to\infty} \frac{a^x - b^x}{c^x - d^x}$

(a) ∞ (b) 0 (c) $\log\frac{a}{b}$ (d) $\frac{\log(a/b)}{\log(c/d)}$

21. $\lim_{x\to 0} \frac{x}{\tan\ x}$is

(a) 1 (b) -1 (c) 0 (d) ∞

22. The 'c' of Lagranges Mean value Theorem for the function $f(x) = x^2 + 2x - 1$; $a = 0$, b = 1 is

(a) -1 (b) 1 (c) 0 (d) $\frac{1}{2}$

23. If $u = x^y$ then $\frac{\partial u}{\partial x}$ is equal to

(a) yx^{y-1} (b) $u \log x$ (c) $u \log y$ (d) xy^{y-1}

24. The curve $y^2 (x - 2) = x^2(1 + x)$ has

(a) An asymptote parallel to x – axis (b) an asymptote parallel to y – axis
(b) asymptotes parallel to both axes (d) no asymptotes

25. $\int_0^\infty x^5\, e^{-4x}$ dx is

(a) $\frac{6\,!}{4^6}$ (b) $\frac{6\,!}{4^5}$ (c) $\frac{5\,!}{4^6}$ (d) $\frac{5\,!}{4^5}$

26. The area between the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and its auxillary circle is

(a) πb(a-b) (b) 2πa(a-b) (c) πa(a-b) (d) $2\pi b$(a-b)

27. The length of the curve $\frac{x^2}{3}+\frac{y^2}{3} = 4$ is

(a) 48 (b) 24 (c) 12 (d) 96

28. The area bounded by the curve x = g(y) to the right of y – axis and two the lines y = c and y = d is given by

(a) $\int_c^d x\, dy$ (b) $\int_a^c x\, dy$ (d) $\int_c^d y\, dy$ (d) $\int_c^d x\, dy$

29. $p \leftrightarrow q$ is equivalent to

(a) $p \to q$ (b) $q \to p$ (c) $(p \to q) \vee (q \to p)$ (d) $(p \to q) \wedge (q \to p)$

30. The order of [7] in $(z_9, +_9)$ is

(a) 9 (b) 6 (c) 3 (d) 1

31. In the group (G, .) G = { 1, -1 , i, -i} order of -*i* is
(a) 2 (b) 0 (c) 4 (d) 3
32. The set of positive even integers, with usual multiplication forms
(a) A finite number (b) only a semi group (c) only a monoid (d) an infinite group
33. In a Poisson distribution $P(X = 0) = k$ then the variance is
(a) $log\frac{1}{k}$ (b) $log\ k$ (c) e^{λ} (d) $\frac{1}{k}$
34. A box contains 6 red and 4 white balls. If 3 balls are drawn at random, the probability of getting 2 white balls without replacement is
(a) $\frac{1}{20}$ (b) $\frac{18}{125}$ (c) $\frac{4}{25}$ (d) $\frac{3}{10}$
35. For a standard normal distribution the mean and variance are
(a) μ , σ^2 (b) μ , σ (c) 0, 1 (d) 1, 1
36. The distribution function $F(x)$ of a random variable X is
(a) A decreasing function (b) a non-decreasing function (d) a constant function
(c) increasing first and then decreasing
37. Solution of $\frac{dx}{dy}$ + mx = 0 where m < 0 is
(a) $x = ce^{my}$ (b) $x = ce^{-my}$ (c) $x = my + c$ (d) $x = y$
38. A particular integral of $(D^2 - 4D + 4)y = e^{2x}$ is
(a) $\frac{x^2}{2}e^{2x}$ (b) x^2e^{2x} (c) xe^{2x} (d) $\frac{x}{2}e^{2x}$
39. The order and degree of the differential equation $y' + (y'')^2 = (x + y'')^2$
(a) 1, 1 (b) 1, 2 (c) 2, 1 (d) 2, 2
40. If cos$x$ is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ then P
(a) - cot x (b) cot x (c) tan x (d) - tan x

SECTION – B **10 x 6 = 60**

(i) Answer any 10 questions. (ii) Question no.55 is compulsory and chooseany 9 questions from the remaining. (iii) Each question carries 6 marks.

41. Solve by matrix determinant method $x + y + 2z = 4$, $2x + 2y + 4z = 8$, $3x + 3y + 6z = 10$
42. (i) Find the value of λ if the points (3, 2, -4), (9, 8, -10) and (λ, 4, -6) are collinear.
(ii) For any vector $\vec{r}$ prove that $\vec{r} = (\vec{r}\cdot\vec{i})\vec{i} + (\vec{r}\cdot\vec{j})\vec{j} + (\vec{r}\cdot\vec{k})\vec{k}$
43. Show that the two lines $\vec{r} = (\vec{i} - j) + t(2\vec{i} + \vec{k})$ and $\vec{r} = (2\vec{i} - j) + s(\vec{i} + \vec{j} - \vec{k})$ are skew lines find the distance between them.
44. Find the square root of $(-8 - 6i)$
45. Prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n+2}{2}}\cos\frac{n\pi}{4}$, $n \in N$
46. Find the equation of the hyperbola if the asymptotes are $2x + 3y - 8 = 0$ and $3x - 2y + 1 = 0$ and (5, 3) is a point on the hyperbola.
47. Obtain the Maclaurin's series for $log_e(1 + x)$

48. Find the absolute maximum and minimum values of the function $f(x) = x^3 - 3x^2 + 1$, $-\frac{1}{2} \leq x \leq 4$
49. If w = x + 2y + z^2 and x = cos$t$; y = sin t ; z = t. Find $\frac{dw}{dt}$
50. Evaluate $\int_0^3 \frac{\sqrt{x}\ dx}{\sqrt{x}+\sqrt{3-x}}$
51. Solve $D^2y = -9 \sin 3x$
52. Show that $\sim (p \vee q) \equiv ((\sim p) \wedge (\sim q))$
53. (i) Prove that identity element of a group is unique.
    (ii) Show that $(a^{-1})^{-1} = a \forall a \epsilon$ G, in a group G.
54. A discrete random variable Xhas the following probability distributions.

X	0	1	2	3	4	5	6	7	8
P(x)	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Find the value of a (ii) Find P(x< 3) (iii) Find P(3 <x< 7)

55. (a) 20% of the bolts produced in a factory are found to be defective. Find the probability that in a sample of 10 bolts chosen at random exactly 2 will defective using (i) Binomial distribution (ii) Poisson distribution [e^{-2} = 0.1353]**[OR]**
    (b) For A = $\begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$, show that $A = A^{-1}$. State and prove reversal law for inverse of matrices.

SECTION – C **10 x 10 = 100**

(i) Answer any 10 questions. (ii) Question no.70 is compulsory and choose any 9 questions from the remaining. (iii) Each question carries 10 marks.

56. Discuss the solutions of the system of equations for all values of λ.
    $x + y + z = 2$, $2x + y - 2z = 2$, $\lambda x + y + 4z = 2$
57. Prove that cos (A + B) = cos A cos B – sin A sin B
58. Find the vector and Cartesian equations of the plane passing through the points (-1, 1, 1) and (1, -1, 1) and perpendicular tothe plane x + 2y +2z = 5
59. The ceiling in a hallway 20ft wide is in the shape of a semi ellipse and 18 ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12 ft.
60. Find the eccentricity, centre, foci and vertices of the hyperbola$9x^2 - 16y^2 + 36x + 32y + 164 = 0$ and also trace the curve.
61. Find the separate equations of the asymptotes of the hyperbola
    $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$
62. Prove that the sum of the intercepts on the co-ordinate axes of any tangent to the curve $x = a \cos^4\theta, y = a \sin^4\theta$, $0 \leq \theta \leq \frac{\pi}{2}$ is equal to a.
63. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ (volume of the sphere)

64. Use differentials to find an approximate value for the given number
$y = \sqrt[3]{1.02} - \sqrt[4]{1.02}$
65. Show that the surface area of the solid obtained by revolving the arc of the curve
$y = \sin x$ from $x = 0$ and $x = \pi$ about x- axis is $2\pi[\sqrt{2} + \log(1 + \sqrt{2})]$
66. Find the common area enclosed by the parabola $4y^2 = 9x$ and $3x^2 = 16y$
67. Show that the set G of all positive rationalsforms a group under the composition $*$
defined by $a*b = \frac{ab}{3}$ for all $a, b \in$ G
68. The sum of Rs.1000 is compound continuously, the nominal rate of interest being four percent per annum. In how many years will the amount be twice the original principal? ($\log_e 2$= 0.6931)
69. Find c, μ and σ^2 of the normal distribution whose probability function is given by
$f(x) = ce^{-x^2+3x}$, $-\infty < X < \infty$
70. (a) P represents the variable complex number z. Find the locus of P, if
$Im\left[\frac{2z+1}{iz+1}\right] = -2$ **[OR]**
(b) Solve $(D^2 - 6D + 9) = x + e^{2x}$

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