X - MATHEMATICS BY EASY AND SHORT CUT METHOD

Here all the problems are worked out by different method from text book methods. All these methods are from CBSE, ICSE VEDIC MATHS, NEPAL COUNTRY TEXT BOOKS. I hope this may be helpful to the student very much. Here more explanations are given for easy understanding and these explanations need not be written in examination or test. Suggestions are most welcome for the better understanding.

10th MATHEMATICS BY EASY AND SHORTCUT METHOD

BY
K. RAVI MADESWARAN,
M.Sc., M.Phil., M.A., M.A., M.Ed., P.G.D.V.E&S
BT. ASSISTANT,
GOVERNMENT HIGHER SECONDARY SCHOOL,
VALLIPATTU, VANIYAMBADI - 635 752.
VELLORE DISTRICT.

Example 2.6

In a flower garden, there are 23 rose plants in the 1st row, 2nd row contains 21 rose plants 3rd row contains 19 rose plants and so on. There are 5 rose plants in the last row. How many rows are there in the flower garden.

The rows are arranged as

23, 21, 19, ………..5 arrange this in reverse order as 5,7,9,…..19,21,23

Since the difference is 2, start the sequence with 2 by subtracting 3 from each i.e. 2,4,6,8,…………20.

From this easily we can understand that the sequence contains 10 rows.

Example 2.7

A person joins his work in 2010 with annual salary of Rs.30,000, and receives an annual increment of Rs. 600 every year in which year his annual salary be Rs.39,000.

His annual salary scheme is in A.P. i.e.

30,000, 30,600, 31,200, ……..39,000.

His increment after 1st year is Rs.600, 2nd year is Rs.1,200, 3rd year is Rs.1800 and on the last year is Rs.9,000.

Here the difference is 600. Hence

$\frac{9,000}{600}$ = 15, After 15 years, i.e. on 2025 his income will be 39,000.

Exercise 2.2-Qn.no.5

Find the 17th Term of the A.P. 4,9,14,…..

The given series is 4,9,14……

Here the difference is 5.

Hence start the series with 5, by adding 1 to all the terms

we get 5,10,15,………

Since the series is multiple of 5, the 17th term is 17*5=85.

Now subtract 1 from 85 which we have added

i.e. 17th term is 84.

Exercise 2.2-Qn.no.6

How many terms are there in the following A.P. 7,13,19,……205.

Here the difference is 6. Hence start the series with 6 by subtracting 1 from each. i.e. 6,12,18,……….204.

Divide last term by 6, weget 204/6=34.

There fore the series contains 34 terms.

Exercise 2.2-Qn.no.11

A television manufacturer produces 1000 television on the 7th year and 1450 on the 10th year. Assuming that the production increases uniformly by a fixed number every year, find the number of TVs produced in the first year and in the 15th year..

7th year =1000
10th year=1450. hence the difference is 3year=450.
Therefore for one year = 150.
On first year = 1000 – (6year production 6*150)
= 1000-900
= 100
On 15th year = 100+ 14*150
=100 + 2100
=2200

Exercise 2.2-Qn.no.12

A man has saved Rs.640, during the 1st month, Rs.720 in the 2nd month, Rs.800 in the 3rd month. If he continues his savings in this sequence, what will be his savings in the 25th month?

Given 640,720,800,……
Here the difference is 80, Hence start the series with 80, by subtracting 560 from each term.
i.e. 80,160,240, ……… 25*80 = 2000
Now add 560 to the last term which we have subtracted.
Therefore the last term is 2560.

Exercise 2.2-Qn.no.16

A person has deposited Rs.25,000 in an investment which yields 14% simple interest annually. Do these amounts (principal+interest) form an A.P.? If so, determine the amount of investment after 20 years.

S.I. = PNR/100
= 25,000 * 1*14 /100
= 3500.
For 1 year the interest is Rs.3,500,
For 2 year the interest is Rs. 7,000. and so on.
For 20 year the interest is 20*3,500 = 70,000.
Therefore the total sum is 70,000 + 25,000 = Rs.95,000.

Example 2.16

Find the sum in A.P. 5+11+17+…….+95

Since the series is in A.P. with difference d=6, add 1 to all terms so that the series will be in multiples of 6

Therefore the series is 6+12+18+……..96.

Here the series contains 96/6=16 terms.

To find the sum take 6 common

$\therefore 6(1+2+3+\ldots\ldots+16) = 6\frac{n(n+1)}{2} - 16$

$= \frac{6*16*17}{2} - 16$

$= 816 - 16 = 800$ (Here we subtract 16 which we have added 1 to each in earlier to make multiple of 6.)

Example 2.20

Find the sum of all the 3 digit numbers which are divisible by 8.

Three digit numbers which are divisible by 8 are 104,112,120,……….992.

To make the problem easier add 4 to each.

i.e. 108,116,124,……….996. and subtract 100 from each

so that the series is 8,16,24,………….896.

Now the number of terms is 896/8 = 112.

To find the sum take 8 common

$\therefore 8(1+2+3+\ldots\ldots+112) = 8\frac{n(n+1)}{2} + 112*100 - 112*4$

$= 8\frac{112*113}{2} + 112*100 - 112*4$

$= 61376$ (here we add 112*100, because we have subtracted 100 from each number and we subtract 112*4. since we added 4 to each number.

Exercise 2.4-Qn.no.3 (i)

Find the sum of 38+35+32+…….+2

Write them in reverse order with d=3

2+5+8+………+32+35+38

Since this series is A.P. with d=3, so add 1 to each term to make multiple of 3

Therefore the series becomes 3+6+9+……+39

$$\text{Sum} = 3(1+2+3+\ldots\ldots+13)$$

$$= 3\frac{n(n+1)}{2} - 13$$

$$= \frac{3*13*14}{2} - 13$$ (here we subtract 13 because we have added 1 to each term.)

$$= 260.$$

Exercise 2.4-Qn.no.14

A sum of Rs.1000 is deposited every year at 8% simple interest. Calculate the interest at the end of each year. Do these interest amounts form an A.P.? If so, find the total interest at the end of 30 years.

The sum of the interest for 30 years is given by 80+160+240+……….+2400

$$= 80(1+2+3+\ldots\ldots\ldots\ldots+30)$$

$$= 80\frac{n(n+1)}{2}$$

$$= \frac{80*30*31}{2}$$

$$= 37,200.$$

Exercise 2.6-Qn.no.1(iv)

Find the sum of 7+14+21+………490

$$=7(1+2+3+\ldots\ldots\ldots+70)$$

$$= 7\frac{n(n+1)}{2}$$

$$= \frac{7*70*71}{2}$$

$$= 17,395$$

Example 3.17

Factorize $2x^3 - 3x^2 - 3x + 2$ in to linear factors.

2-3-3+2≠ 0 therefore (x-1) is not a factor.

2 -3 = -3+2 therefore (x+1) is a factor.

$2x^3 - 3x^2 - 3x + 2 = (x+1)(2x2 \quad +2)$. here divide $2x^3$ by x to get $2x^2$ now to get the middle term multiply the coefficient of x^3 by -1 i.e. 2* -1 = -2 and add with the coefficient of x^2. we get -5 as the coefficient of x . divide last term by 1 to get 2.

$2x^3 - 3x^2 - 3x + 2 = (x+1)(2x^2 - 5x + 2)$

Now let us see how to factorize $2x^2 - 5x + 2$ by short cut method.

Divide the middle term into two terms such that the ratio of first two terms is equal to the ratio of last two terms. i.e. 2:4 = 1:2

We write term as $2x^2 - 4x$ <u>– x +2</u>

Now take - common from the last two terms i.e. –(x-2)

$2x^2 - 5x -+2 = (x-2)(2x - 1)$

i.e. divide $2x^2$ by x toget 2x and divide 2 by – 2 to get -1

Therefore $2x^3 - 3x^2 - 3x + 2 = (x+1)(x-2)(2x - 1)$

Exercise 3.5-Qn.no.1(i)

Factorize $x^3 - 2x^2 - 5x + 6$

Sum of the co-efficients 1-2-5+6 = 0

Therefore (x-1) is a factor. By the above process

$x^3 - 2x^2 - 5x + 6 = (x - 1)(x^2 - x - 6)$ i.e. divide x^3 by x to get x^2 and multiply coefficient of x^3 by +1 and addwith thecoefficient of x^2 to get the middle term. And divide last term 6 by the last term of (x – 1). i.e. by -1.

Again factorize $(x^2 - x - 6)$

$x^3 - x^2 - 5x + 6 = (x-1)(x - 3)(x+2)$

Exercise 3.5-Qn.no.1(ii)

Factorize $x^3 - 23x^2 + 142x - 120$

1-23+142-120=0 Therefore (x-1) is a factor.

$x^3 - 23x^2 + 142x - 120 = (x - 1)(x^2 - 22x + 120)$ i.e. Divide x^3byx to get x^2 and

$= (x-1)(x-10)(x-12)$ Multiply coefficient of x^3 By +1 and add with -23 to get -22 and divide -120 by -1 to get 120

Exercise 3.5-Qn.no.1(iv)

Factorize $4x^3 – 5x^2 + 7x - 6$

4-5+7-6=0 Therefore (x-1) is a factor.

$4x^3-5x^2+7x-6 = (x-1)(4x^2 –x+6)$ (Here divide $4x^3$ by x to get $4x^2$ and multiply coefficient of x^3, 4 by +1 to get 4 and add with -5. we get -1 i.e. –x and divide the last term -6 by -1

Exercise 3.5-Qn.no.1(x)

Factorize $2x^3 + 11x^2 – 7x – 6$

2+11-7-6 =0 Therefore (x-1) is a factor.

2x3+11x2-7x-6 = $(x-1)(2x^2+13x+6)$ by following above procedure.

Now let us see how to factorise $2x^2+13x+6$ by easy way.

Split the middle term in to two terms so that ratio between 1st and 2nd is equal to 3rd and 4th i.e. $2x^2 : 12x :: x : 6$ Now take the common term from x+6 i.e. 1. and divide first term of $2x^2+13x+6$ by x to ge 2x and last term by 6 to get 1

Therefore $2x^3+11x^2-7x-6 = (x-1)(x+6)(2x+1)$

Example 3.20

Find the GCD of the polynomials x^4+3x^3-x-3 and x^3+x^2-5x+3

Let $x^4+3x^3+0x^2-x-3$ …………..(1)

x^3+x^2-5x+3………......(2)

(1) – (2)x $=(x^4+3x^3+0x^2-x-3) – (x^4+x^3-5x^2+3x)$

$= 2x^3+5x^2-4x -3$ …………..(3)

(3) + (2) $= (2x^3+5x^2-4x -3) + (x^3+x^2-5x+3)$

$= 3x^3+6x^2-9x$

$= 3x(x^2+2x-3)$

Therefore $(x^2+2x-3)$is the GCD.

Example 3.21

Find the GCD of the polynomials $3x^4+6x^3-12x^2-24x$ and $4x^4+14x^3+8x^2-8x$.

This GCD can be found by **addition and subtraction method**.

Multiply the first term by 4 …$12x^4+24x^3-48x^2-96x$…..(1)

Multiply the 2nd term by 3 … $12x^4+42x^3+24x^2-24x$ …(2)

(1) – (2) = $-3x^3-12x^2-12x$

$= -3x(x^2+4x+4)$

GCD is $x(x^2+4x+4)$

Exercise 3.6-Qn.no.3(i)

Find GCD of the following pairs of polynomials using division algorithm**.**

$x^3-9x^2+23x-15$, $4x^2-16x+12$

Let $x^3-9x^2+23x-15$ …………….(1)

$4x^2-16x+12$ ………………….(2)

Here higher powers are removed by making them equal . Therefore Multiply equation (2) by x and divide by4 we get x^3-4x^2+3x……… (3)

(1) – (3) =$-5x^2+20x-15$

$= -5(x^2-4x+3)$ Therefore GCD is (x^2-4x+3)

Exercise 3.6-Qn.no.3(ii)

$3x^3+18x^2+33x+18$, $3x^2+13x+10$

Multiply the second term by x for the removal of higher power.

$(3x^3+18x^2+33x+18) – (3x^3+13x^2+10x)$

$= 5x^2+23x+18$………(1)

Let $3x^2+13x+10$…….(2)

Multiply (1) by 3 and (2) by 5and subtract.

$(15x^2+69x+54) – (15x^2+65x+50) = 4x+4$

$= 4(x+1)$

Therefore GCD is x+1.

Exercise 3.6-Qn.no.3(iii)

$2x^3+2x^2+2x+2$, $6x^3+12x^2+6x+12$

Take 2 common from the 1st term and 6 from the 2nd term and subtract.

$(x^3+x^2+x+1) – (x^3+2x^2+x+2) = -x^2-1$
$= -(x^2+1)$ and common factor of 2 and 6 is 2.

Therefore GCD is $2 (x^2+1)$

Exercise 3.6-Qn.no.3(iv)

$x^3-3x^2+4x-12$, $x^4+x^3+4x^2+4x$

Take x common from 2nd term and subtract from 1st term.
$(X^3-3x^2+4x-12) – (x^3+x^2+4x+4)$
$= -4x^2-16$
$= -4(x^2+4)$ Therefore GCD is (x^2+4)

Exercise 3.10-Qn.no.2(vii)

Simplify the following
Divide $\frac{2x^2+5x-3}{2x^2+9x+9}$ by $\frac{2x^2+x-1}{2x^2+x-3}$

Let us factorize these terms by shortcut method.
$2x^2+5x-3$ Here split the middle term into two terms such that ratio of first two terms equal to last two.
i.e. $2x^2+6x-x-3 =(x+3)(2x-1)$
Take – from last term $–x-3 =(x+3)$
Now Divide first term by first and last term by last term
i.e. $2x^2/x$ and $-3/3$ Which gives $(2x-1)$
By the same method $2x^2+9x+9 = 2x^2+6x+\underline{3x+9}$ Take3 common
$= (x+3)(2x+3)$
$2x^2+x-1 = 2x^2+2x\underline{- x -1}$ Take (-) common
$= (x+1)(2x -1)$

Exercise 3.12-Qn.no.2(v)

Find the square root of
$(6x^2+5x-6)(6x^2-x-2)(4x^2+8x+3)$ Factorize using the above method

$6x^2+5x-6 = 6x^2+9x-4x-6 = (2x+3)(3x-2)$
$6x^2-x-2 = 6x^2-4x+3x-2 = (3x-2)(2x+1)$
$4x^2+8x+3= 4x^2+6x+2x+3= (2x+3)(2x+1)$

Therefore the square root is $(2x+3)(3x-2)(2x+1)$

Example 3.33

Find the square root of $x^4-10x^3+37x^2-60x+36$

$x^2 -5x +6$

x^2	$x^4 – 10x^3 + 37x^2 – 60x +36$
$+ x^2$	x^4
$2x^2 -5x$	$-10x^3 + 37x^2$
$-5x$	$-10x^3 + 25x^2$
$2x^2-10x +6$	$12x^2 – 60x + 36$
	$12x^2 – 60x + 36$
	0

Here we add the quotient instead of multiplying by 2.

In Fifth Unit, Area of the triangle = 1/2{$(x_1y_2+x_2y_3+x_3y_1)- (y_1x_2+y_2x_3+y_3x_1)$}
Area of the Quadrilateral = ½ {$(x_1y_2+x_2y_3+x_3y_{4+X_4Y_1})- (y_1x_2+y_2x_3+y_3x_{4+Y_4X_1})$}

Example 5.22

Find the equation of the straight line passing through the point (-1,1) and (2,-4)
$x_1=-1$, $y_1=1$, $x_2=2$, $y_2=-4$
Two point formula is $\mathbf{(y_2-y_1)x – (x_2-x_1)y = (y_2-y_1)x_1 – (x_2-x_1)y_1}$
$(-4-1)x – (2+1)y = -5*-1 – 3*1$
$-5x -3y = 5-3$
$-5x-3x = 2$ This gives $5x+3y+2=0$

ANGLE BISECTOR THEOREM

The internal (external) bisector of an angle of a triangle divides the opposite side internally (internally) in the ratio of the corresponding sides containing the angle.

Case (i) Internally
Given : In ABC, AD is the internal bisector of
BAC which meets BC at D.
To Prove: $\frac{BD}{DC} = \frac{AB}{AC}$

Constrcution: Draw CE II DA to meet BA produced at E.
Let ∟1= ∟BAD, ∟2= ∟DAC, ∟3= ∟ACE, ∟4= ∟AEC, Diagram can be copied from the text book

∟1= ∟4 Corresponding angles are equal (1)
∟2= ∟3 Alternate angles are equal (2)
∟1= ∟2 AD is a angle bisector. (3)
→ ∟3= ∟4
ΔACE is an isosceles
AE=AC
Noe in ΔBCE we have CE II DA

$$\frac{BD}{DC} = \frac{BA}{AE} \quad \text{(Thales theorem)}$$

$$\frac{BD}{DC} = \frac{AB}{AC} \quad \text{(AE=AC)}$$ Hence the theorem.

Similar method can be followed for external angle bisector. (not for exam)

PYTHAGORAS THEOREM

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: In a right angled ΔABC, ∟A=90°

Diagram can be copied from the text book

To prove: $BC^2 = AB^2 + AC^2$

Construction: Draw AD ⊥ BC.

ΔABD ˜ ΔABC

The Areas of the similar triangles are propotional to the square of their sides.

$$\frac{\Delta ABD}{\Delta ABC} = \frac{AB^2}{BC^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

similarly ΔAD C ˜ ΔABC

$$\frac{\Delta ADC}{\Delta ABC} = \frac{AC^2}{BC^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

(1)+(2) gives

$$\frac{AB^2}{BC^2} + \frac{AC^2}{BC^2} = \frac{\Delta ABD}{\Delta ABC} + \frac{\Delta ADC}{\Delta ABC}$$

$$= \frac{\Delta ABC}{\Delta ABC} = 1$$

$$AB^2 + AC^2 = BC^2$$

HIGHTS AND DISTANCE

When two objects are in angle of elevation

$$h = \frac{a}{\tan\alpha - \tan\beta} \quad \ldots\ldots\ldots\text{I}$$

h= hight of the tower
a= distance between two objects

When two objects are in angle of depression on same side

$$h = \frac{a}{\cot\alpha - \cot\beta} \quad \ldots\ldots\ldots\ldots\text{II}$$

When two objects are in angle of depression on either side

$$h = \frac{a}{\cot\alpha + \cot\beta} \quad \ldots\ldots\ldots\ldots\text{III}$$

Example 7.19

A jet fighter at a height of 3000 m from the ground, passes directly over another jet fighte at an instance when their angles of elevation from the same observation point are 60° and 45° respectively. Find the distance of the first jet fighter from the second jet at that instant.

$$h = \frac{a}{\tan\alpha - \tan\beta} \quad \ldots\ldots\ldots\text{I}$$

h= hight of the tower
a= distance between two objects

$$3000 - h = \frac{h}{\tan 60° - \tan 45°}$$

$$3000 - h = \frac{h}{\sqrt{3} - 1}$$

h= 1268 m

Example 7.21

A vertical wall and a tower are on the ground. As seen from the top of the tower, the angles of depression of the top and bottom of the wall are 45° and 60° respectively. Find the height of the wall if the height of the tower is 90m.

$$h = \frac{a}{\cot\alpha - \cot\beta}$$

$$90 = \frac{a}{\cot 45^\circ - \cot 60^\circ}$$

$a = 38.04m$

Example 7.22

A girl standing on a lighthouse built on a cliff near the seashore, observes two boats due East of the lighthouse. The angles of depression of the two boats are 30° and 60°. The distance between the boats is 300m. Find the distance of the top of the lighthouse from the sea level.

$$h = \frac{a}{\cot\alpha - \cot\beta}$$

$$h = \frac{300}{\cot 30^\circ - \cot 60^\circ}$$

$$h = \frac{300}{\sqrt{3} - 1/\sqrt{3}}$$

$h = 150\sqrt{3}$ m

Note Examples 7.23, 7.24, 7.26, Can be worked out using the formula I
Exercise 7.2 Problem numbers 8 can be done using the formula III
Exercise 7.2 Problem numbers 10, 11,12,13, 14, 15, can be done using the formula II.

Diagram for concern problems can be copied from the text book.